Grey wolf optimizer: Overview, modifications and applications

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ABSTRACT

The complexity of real-world problems motivated researchers to innovate efficient problem-solving techniques. Generally natural Inspired, Bio Inspired, Metaheuristics based on evolutionary computation and swarm intelligence algorithms have been frequently used for solving complex, real-world optimization and Non-deterministic polynomial hard (NP-Hard) problems because of their ability to adjust to a variety of conditions. This paper describes Grey Wolf Optimizer (GWO) as a Swarm Based metaheuristic algorithm inspired by the leadership hierarchy and hunting behavior of the grey wolves for solving complex and real-world optimization problems. Since the appearance of GWO many modifications for improving the performance of the algorithm and have been applied to various applications in several fields. At the end of this paper, the improvements are listed.

INTRODUCTION

A metaheuristic is a collection of algorithmic frameworks inspired by nature designed to provide the fittest near-optimal solution for optimization problems (Marqas et al., 2020). These algorithms are preferred especially when the available information is not complete and computational capacities are limited (Bianchi et al., 2009). A metaheuristic is different from a heuristic as it is not application-specific and may be applied for a variety of problems with a few assumptions. These algorithms are applied when the solution space is too large to be completely sampled (Almufti et al., 2021a). Metaheuristics algorithms are divided into four categories as shown in Fig 1. This paper presents the Grey Wolf Optimizer (GWO) algorithm, which is a Swarm-Based algorithm that belongs to the Nature-inspired category of Metaheuristics algorithms.

Grey Wolf Optimizer (GWO) developed by Mirjalili et al. in 2014 (Mirjalili et al., 2014). The GWO is a metaheuristic algorithm that belongs to the third category (Nature-inspired). This algorithm is inspired by the hunting process found in Grey Wolves. This is unique as it follows the leadership hierarchy of the grey wolves. Grey wolves are well known for pack hunting and no other SI methods were proposed to follow this hierarchical hunting behavior (Almufti, 2018). This paper is an attempt to review the GWO algorithm and its proposed applications which help the researcher in the future to apply it in other new and promising fields of application such as path planning and path following control of autonomous underwater vehicles.

Metaheuristics Algorithms in the past the methods that have a stochastic mechanism were often called a heuristic algorithm, nowadays in recent studies, it refers to as metaheuristics which is a combination of two words Meta and Heuristic, where the world "heuristic" means finding or discovering a goal by trial and error, and the world "meta" means a beyond or higher level, that means a metaheuristic generally refers a "higher level of heuristics" (Almufti, 2018; Glover, 1986). Generally, metaheuristic algorithms represent a "master strategy that guides and modifies other heuristics to produce solutions beyond those that are normally generated in a quest for local optimality" (Almufti, 2019a; Ihsan et al., 2021). Those algorithms use a certain adjustment of randomization and local search. A good solution for difficult optimization problems can be found in a reasonable time, but in general, there is no guarantee of finding optimal solutions (Almufti, 2018; Marqas et al., 2020; Marashdih et al., 2018). In the fields of computer science, mathematical optimizations, and engineering's, the term "metaheuristic" represents a higher-level procedure or heuristic designed to search, find, generate, or select a heuristics, that may provide a good solution to an optimization problem, especially for the large problems such as (NP-hard problem) or in case of limit, incomplete or imperfect information (Asaad et al., 2018; Saban et al., 2018; Yahya et al., 2020). Metaheuristics consist of a set of solutions that is too large to be completely sampled. Metaheuristics may make few assumptions about the optimization problem being solved, and so they may be usable for a variety of problems. (Balamurugan et al., 2015). Comparing with optimization algorithms or iterative methods, metaheuristics do not guarantee that the best solution globally optimal solution can be found on some class of problems (Mirjalili et al, 2014; Salim et al., 2018). Many metaheuristics implement some form of stochastic optimization so that the solution found is dependent on the set of random variables generated. In combinatorial optimization, by searching over a large set of feasible solutions, metaheuristics can often find good solutions with less computational effort than optimization algorithms, iterative methods, or simple heuristics. As such, they are useful approaches for optimization problems (Marqas et al., 2019; Yahya et al., 2019). Metaheuristics algorithms are categorized into four categories as shown in fig 1. Each of these categories is further sub-divided, in this paper GWO which is a swarm-based algorithm that belongs to the nature-inspired category of metaheuristics algorithms is reviewed (Saeed et al., 2019).
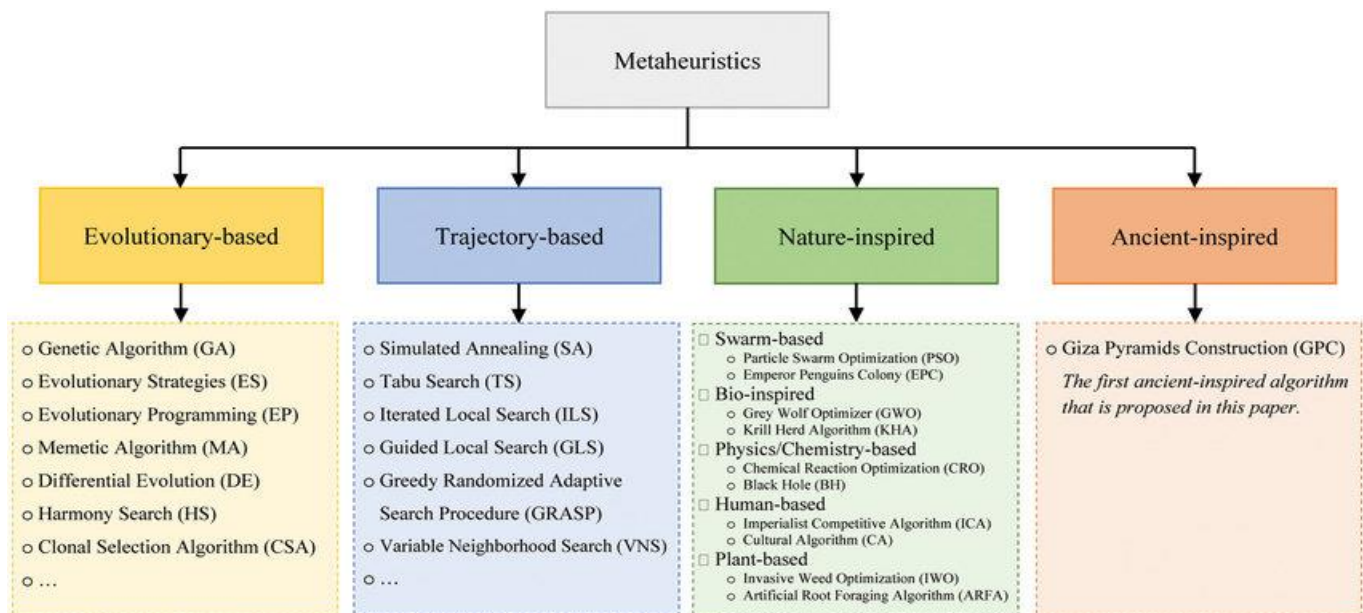
Figure 1. Categories of metaheuristics algorithms (Harifi, S. et al. (2020)

GREY WOLF OPTIMIZER (GWO)

GWO is a swarm intelligence-based meta-heuristic algorithm that mimics the leadership hierarchy and hunting process of grey wolves in nature and mathematically models them (Al-Aboody et al., 2016; Panda et al., 2019). Grey wolves often prefer to live together. The population-based on the importance is divided into four groups which are and, respectively. The alphas are the leaders of the group and are responsible for making decisions for hunting. The second level includes betas that help alphas in making decisions and other group activities. The omegas have the lowest level of hierarchy and are submissive to and wolves. If a wolf is not a or, then it is. The deltas follow a and b wolves but dominate the omega. The most important wolf groups are and, respectively, that should guide omegas into areas with higher hunting probability (Panda et al., 2019; Mirjalili et al., 2015). Figure 2 shows the steps of the GWO Algorithm for solving optimizations problems.
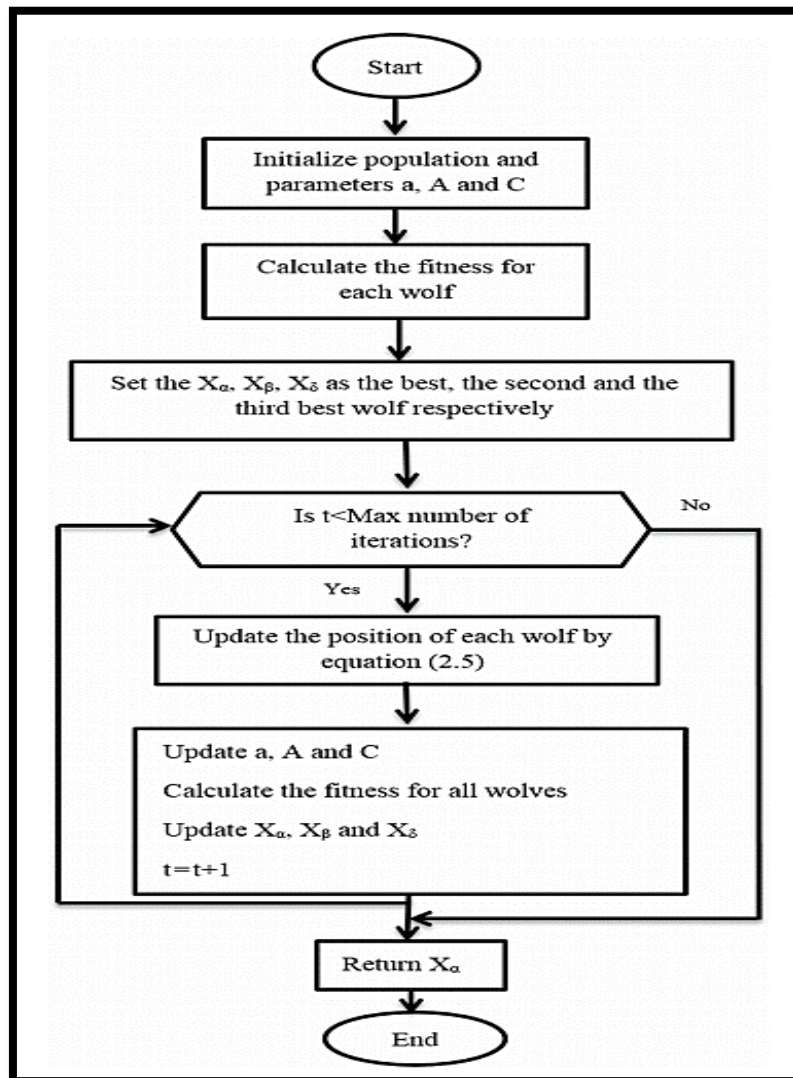
Figure 2. GWO flowchart (Amirsadri et al., 2017)

INSPIRATION OF GWO

GWO is a swarm-based algorithm inspired by the social intelligence of grey wolf leadership and hunting strategies. Generally, the grey wolf has four packs (Alpha, Beta, Delta, and Omega). In each pack of grey wolves, there is a common social hierarchy that dictates power and domination (see Figure. 3). In the Gray wolf hierarchy strategy, the most powerful wolf is alpha leads the entire pack in hunting, migrations, and feeding processes. In case of the absence of alpha wolf from the pack, the strongest wolf of the beta pack wolfs takes the lead of the pack (Faris et al., 2017; Almufti, 2017). The power and domination of the two other packs delta and omega are lower than alpha, and beta as can be seen in Fig. 3. This social intelligence is the main inspiration of the GWO algorithm. Another inspiration is the hunting approach of grey wolves. When hunting prey, grey wolves follow a set of efficient steps: encircling and attacking. This allows them to hunt big prey (Almufti, 2017; Negi et al., 2020).
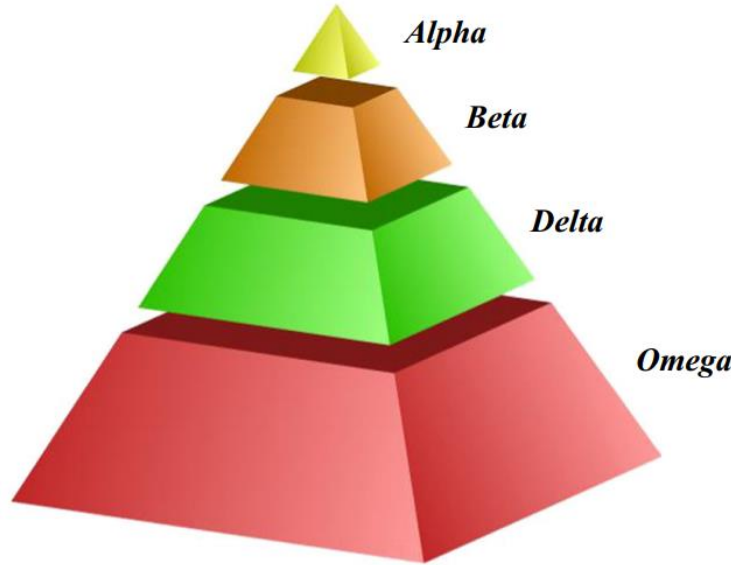
Figure 3. Social hierarchy of grey wolves (Mirjalili et al., 2015)

MATHEMATICAL FORMULATED OF GWO

GWO algorithm divide into two phases includes:

- Encircling
- Attacking (Hunting)

a. Encircling

As encircling phase, the first step in hunting process to chase and encircle. Mathematically in this phase for an n-dimensional space GWO considers two wolfs (points) and it updates their location of the first one based on that of the second one. Based on Eq.1:

$$X(t + 1) = X(t) - A.D \tag{1}$$

Where $X(t + 1)$ represents the next location of the wolf, $X(t)$ represent the current location, A is a coefficient matrix and D is a vector that depends on the location of the prey (Xp) and is calculated as show in eq.2.

$$D = |C.X_P(t) - X(t)| \tag{2}$$

Where C can be calculated by eq. 3.

$$C = 2 . r_2 \tag{3}$$

Where r_2 is a randomly generated vector $\in [0,1]$. By using these two equations, a solution is able to relocate around another solution. Note that the equations use vectors, so this is applied to any number of dimensions. An example of possible positions of a grey wolf with respect to a prey is shown in Figure. 4.
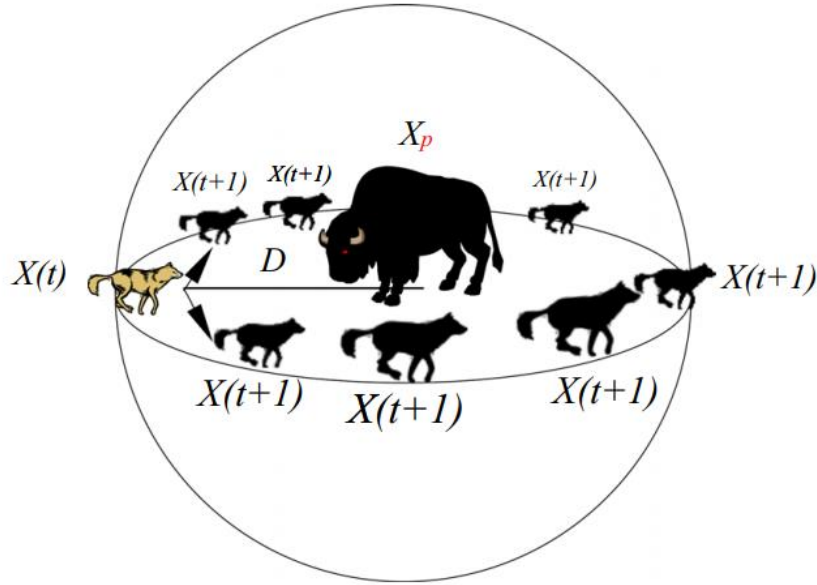
Figure 4. Wolfs position updating (Panda, M. & Das, B. (2019)

Note that the random components in the above equations simulate different step sizes and movement speeds of grey wolves. The equations to define their values are as follows:

$$A = 2a \cdot r_1 - a \tag{4}$$

Where a is a vector where its values are linearly decreased from 2 to 0 during the course of run. r1 is a randomly generated vector from the interval [0,1]. The equation to update the parameter a is as follows:

$$a = 2 - t\left(\frac{2}{T}\right) \tag{5}$$

Where t shows the current iteration and T is the maximum number of iterations.

b. *Attacking (Hunting) phase*

With the equations presented above, a wolf can relocate to any point in a hypersphere around the prey. However, this is not enough to simulate to social intelligence of grey wolves. It was discussed above that social hierarchy plays a key role in the hunt and the survival of a pack. To simulate social hierarchy, the three best solutions are considered to be alpha, beta, and delta. Although in nature there might be more than one wolf in each category, it is considered that there is only one solution belong to each class in GWO for the sake of simplicity.
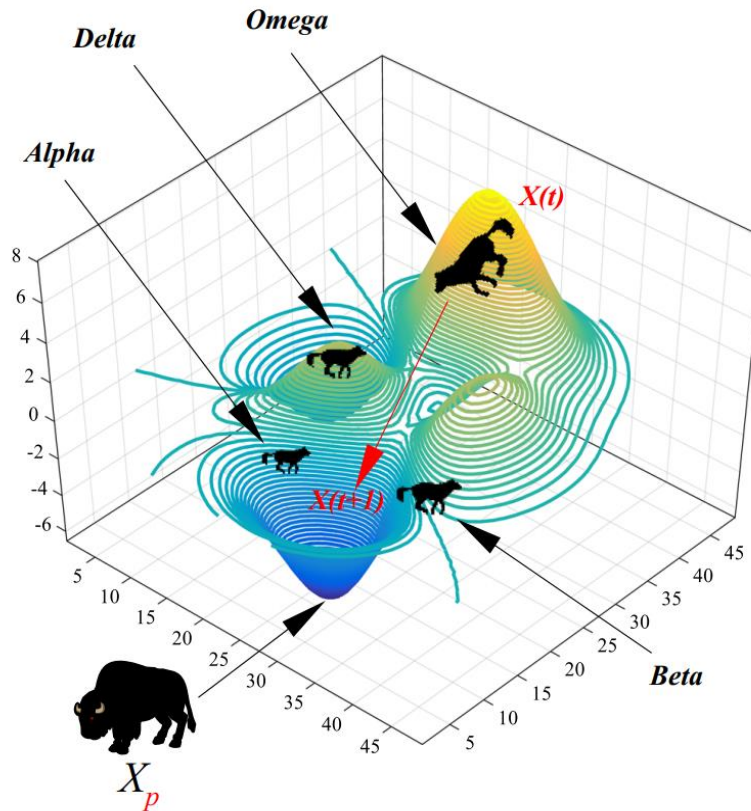
Figure 5. Alpha, beta, delta and omega in GWO (Mirjalili, 2015)

The concepts of alpha, beta, delta, and omega are illustrated in Figure 5. Note that the objective is to find the minimum in this search landscape. It may be seen in this figure that alpha is the closest solution to the minimum, followed by beta and delta. The rest of the solutions are considered omega wolves. There is just one omega wolf in Figure 5, but there can be more. In GWO, it is assumed that alpha, beta, and delta are always the three best solutions obtained so far. The global optimum of optimization problems is unknown, so it has been assumed that alpha, beta, and delta have a good idea of their location, which is reasonable because they are the best solutions in the entire population. Therefore, other wolves should be obliged to update their positions as follows:

$$X(t + 1) = \frac{1}{3}X_1 + \frac{1}{3}X_2 + \frac{1}{3}X_3 \qquad (6)$$

Where X_1 and X_2 and X_3 are calculated with Eq. 7.

$$\begin{aligned} X_1 &= X_\alpha(t) - A_1.D_\alpha \\ X_2 &= X_\beta(t) - A_2.D_\beta \\ X_3 &= X_\delta(t) - A_3.D_\delta \end{aligned} \qquad (7)$$

Where D_α, D_β and D_δ can be calculated by Eq. 8.

$$\begin{aligned} D_\alpha &= |C_1.X_\alpha - X| \\ D_\beta &= |C_2.X_\beta - X| \\ D_\delta &= |C_3.X_\delta - X| \end{aligned} \qquad (8)$$

MODIFICATIONS OF GWO

Generally, all the metaheuristics algorithms after their first appearance undergo many modifications and improvements, so that they can be used to solve various problems (Almufti, 2021b; Wen et al., 2015). After the appearance of the Grey Wolf Optimizer (GWO) algorithm in 2014 (Mirjalili S, Mirjalili SM, Lewis, A., 2014; Almufti, S., 2019) to meet the demands of the real-world problems many modifications have been applied to the original GWO to improve the performances of the proposed algorithm. The modifications were in two various ways: Unhybridized GWO: This form includes the original or the standard GWO as proposed by Mirjalili (Mirjalili et al., 2014) and all the varied forms of GWO which have been modified with different parameters and approaches to the Original GWO along with the different variants (Almufti, 2017). Hybridized GWO: Hybridization means a combination of two or more algorithms to utilize the benefits of the characteristics of each algorithm in the best possible way. The hybridized algorithm showed competitive results in terms of convergence rate, quality results, efficiency, and stability than the individual algorithms in most cases (Almufti, 2017).

In this section, some of the modifications and improvements of the GWO algorithm are listed and arranged according to the development year, as shown in Table (1).

Table 1. GWO based algorithms

#	Abbr.	Name	Author	Type	Year	Ref.
1.	IGWO	Improved GWO	Wen et al.	Unhybridized	2015	(Wen et al.,2015)
2.	CEGWO	Complex-valued encoding GWO	Luo et al.	Unhybridized	2015	(Luo et al., 2015)
3.	BGWO	Binary GWO	Emary et al.	Unhybridized	2016	(Emary et al., 2015)
4.	MGWO	Modified GWO	Mittal et al.	Unhybridized	2016	(Mittal et al., 2016)
5.	MDGWO	Modified discrete GWO	Li et al.	Unhybridized	2016	(Li et al., 2016)
6.	MOGWO	Multi-objective GWO	Mirjalili et al.	Unhybridized	2016	(Marijalili et al., 2016)
7.	EGWO	Enhanced GWO	Joshi and Arora	Unhybridized	2017	(Joshi et al., 2017)
8.	GWO-PSO	GWO- particle swarm optimizer	Singh and Singh	Hybridized	2017	(Singh et al., 2017a)
9.	GWO-ACO	GWO-Ant Colony Optimization	Ab Rashid	Hybridized	2017	(Ab,2017)
10.	GWO-GA	GWO- Genetic Algorithm	Tawhid and Ali	Hybridized	2017	(Tawhid et al., 2017)
11.	GWO-SCA	GWO- Sine Cosine Algorithm	Singh and Singh	Hybridized	2017	(Singh et al., 2017b)
12.	CGWO	Chaotic GWO	Kohli and Arora	Unhybridized	2018	(Kohli et al., 2018)
13.	EEGWO	Exploration-enhanced GWO	Long et al.	Unhybridized	2018	(Long et al., 2018)
14.	INGWO	Intelligent GWO	Liu et al.	Unhybridized	2018	(Liu et al., 2018)
15.	VWGWO	variable weights GWO	Gao and Zhao	Unhybridized	2019	(Gao et al., 2019)
16.	RLGWO	Refraction learning-GWO	Long et al.	Unhybridized	2019	(Long et al., 2019)

GWO APPLICATIONS

Over year, Grey Wolf Optimizer (GWO) algorithm and its modifications showed high-performance in solving different real-world problems and it has been used for solving unconstrained, constrained, multi-objective and NP-Hard problems in fields of engineering, medical, environment, etc. as summarized in Table 2.

Table 2. Summarize GWO applications in different field

#	Application	Discussion	Reference
1.	Feature selection	Feature selection is one of the important processes in machine learning and data mining. The goal of feature selection is to reduce the number of features, select the most representative ones and to eliminate redundant, noisy and irrelevant features. The problem of searching for best set of features is considered as complex and difficult problem due to the extremely large search space when the number of features is large.	(Negi et al., 2020) (Mittal et al., 2016) (Eary et al., 2015) (Vosooghifard et al., 2015) (Mejahed et al., 2016) (Yamany et al., 2016)
2.	Clustering applications	Clustering is a common machine learning and data mining task where the goal is to divide data instances into several groups that have similar characteristics in some sense.	(Kumar et al., 2017) (Zhang et al., 2015) (Yang et al., 2015)
3.	Training neural networks	Artificial neural networks (ANNs) are information processing models inspired by the biological nervous systems. ANNs are widely applied in research and practice due to their high capability for capturing nonlinearity and dynamicity. However, the performance of ANNs is highly affected by their structure and connection weights. Traditionally, the efficiency of any new metaheuristic algorithm is investigated in optimizing the connection weights neural networks shortly after its release.	(Mirjalili, 2015b) (Mosavi et al., 2016) (Mohamed et al., 2015) (Almufti,2021)
4.	Design and tuning controllers	In control engineering, we have noticed an increased number of publications that investigate the application of GWO in tuning the parameters of controllers such as integral (I), proportional-integral (PI), proportional-integral-derivative (PID).	(Li et al., 2015) (Yadav et al., 2016) (Das et al., 2015)
5.	Knapsack Problem	The knapsack problem is a problem in combinatorial optimization: Given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.	(Yassien et al., 2017)
6.	Power dispatch problems	The economic load dispatch (ELD) is a class of non-convex and highly nonlinear constrained optimization problem concern with finding an optimal load dispatch to operate and plan the current resources. This problem is a kind of optimization problem in which its complexity is increased based on the number of system units to be planned. It is concern with distributing the required electricity among the generating units in optimum way in order to minimize the fuel consumptions of each unit in accordance with a power balance equality constraints and power output inequality constraints.	(Wong et al., 2014) (Song et al., 2014) (jayabarathi et al., 2016)
7.	Robotics and path planning	Path-planning is an important primitive for autonomous mobile robots that lets robots find the shortest – or otherwise optimal – path between two points. Otherwise, optimal paths could be paths that minimize the amount of turning, the amount of braking or whatever a specific application requires.	(Zhang et al., 2015) (Tsai et al., 2016) (Korayem et al., 2015)
8.	Wireless sensor network	Generally, GWO had a high-performance in Wireless sensor network applications. It has been used in Tackling the coverage problem in WSN, routing and localization problem in WSNs	(Dao, 2016) (Al-Aboody et al., 2016) (Fouad et al., 2015)

9.	Scheduling	One common scheduling problem is the job shop, in which multiple jobs are processed on several machines. Each job consists of a sequence of tasks, which must be performed in a given order, and each task must be processed on a specific machine. GWO has been used to solve scheduling problem	(Lu et al., 2015) (Lu et al., 2016)
10.	Surface Wave Dispersion Curve	"Surface Wave Dispersion Curve Inversion" scheme based on GWO. The proposed method has been verified on environments both with and without noise and proved to be efficient.	(Song et al., 2015)
11.	Biomedical Research	GWO can also be employed for modelling modular granular neural networks (MGNN). The proposed design performed optimal granulation of data for human recognition	(Sanchez et al., 2017) (Parsian et al., 2017)
12.	NP-hard problem	Generally, GWO had a high-performance in solving NP-Hard problems	(Korayem et al., 2015)

CONCLUSION

Grey Wolf Optimizer (GWO) algorithm is a Swarm-based metaheuristic algorithm proposed in 2014 by Mirjalili et al. After its appearance, many modifications have been proposed on it and it has been adapted to solve the various problem in different fields. This paper firstly addressed this overviewed the original GWO algorithm and then it presented some of its modifications were presented in detail, finally some of its applications considering parameter tuning, different approaches for feature selection and classification, and then hybridized forms were discussed. The applications in different fields were discussed including engineering medical, power dispatch, reliability optimization, etc.

REFERENCES

Ab Rashid, M.F.F. (2017). A hybrid Ant-Wolf Algorithm to optimize assembly sequence planning problem. Assembly Autom 37(2):238–248.

Al-Aboody NA, Al-Raweshidy, H.S. (2016). Grey wolf optimization-based energy-efficient routing protocol for heterogeneous wireless sensor networks. In: 2016 4th international symposium on computational and business intelligence (ISCBI). IEEE, pp 101–107.

Almufti S. (2018). U-Turning Ant Colony Algorithm powered by Great Deluge Algorithm for the solution of TSP Problem, Hdl.handle.net, 2018. [Online].

Almufti, S. (2017). Using Swarm Intelligence for solving NP-Hard Problems. Academic Journal of Nawroz University, 6(3), 46-50.doi: 10.25007/ajnu.v6n3a78.

Almufti, S. (2019). Historical survey on metaheuristics algorithms. International Journal of Scientific World, 7(1), 1.doi: 10.14419/ijsw.v7i1.29497

Almufti, S. (2021). The novel Social Spider Optimization Algorithm: Overview, Modifications, and Applications. Icontech International Journal, 5(2), 32-51. doi: 10.46291/icontechvol5iss2pp32-51

Almufti, S., Marqas, R., Othman, P. & Sallow, A. (2021). Single-based and Population-Based Metaheuristics Algorithms Performances in Solving NP-hard Problems. *Iraqi Journal of Science*.doi: 10.24996/10.24996/ijs.2021.62.5.34.

Amirsadri, S., Mousavirad, S. & Ebrahimpour-Komleh, H. (2017). A Levy flight-based grey wolf optimizer combined with back-propagation algorithm for neural network training. Neural Computing and Applications, 30(12), 3707-3720.doi: 10.1007/s00521-017-2952-5.

Asaad, R. & Abdulnabi, N. (2018). Using Local Searches Algorithms with Ant Colony Optimization for the Solution of TSP Problems. Academic Journal of Nawroz University, 7(3), 1-6. doi: 10.25007/ajnu.v7n3a193.

Balamurugan, R., Natarajan, A. & Premalatha, K. (2015). Stellar-Mass Black Hole Optimization for Biclustering Microarray Gene Expression Data. Applied Artificial Intelligence, 29(4), 353-381. doi: 10.1080/08839514.2015.1016391.

Bianchi, L., Dorigo, M., Gambardella, L. & Gutjahr, W. (2009). A survey on metaheuristics for stochastic combinatorial optimization. Natural Computing, 8(2), 239-287. doi: 10.1007/s11047-008-9098-4

Dao, T.K. (2016). Enhanced diversity herds grey wolf optimizer for optimal area coverage in wireless sensor networks. In: Genetic and evolutionary computing: proceedings of the tenth international conference on genetic and evolutionary computing, November 7–9, 2016 Fuzhou City, Fujian Province, China, vol 536. Springer, p 174.

Das, K.R., Das, D. & Das, J. (2015) Optimal tuning of pid controller using gwo algorithm for speed control in dc motor. In: 2015 international conference on soft computing techniques and implementations (ICSCTI). IEEE, pp 108–112.

Emary E, Zawbaa, H.M. & Hassanien, A.E. (2016). Binary grey wolf optimization approaches for feature selection. Neurocomputing 172:371–381.

Emary, E., Yamany, W., Hassanien, A. & Snasel, V. (2015). Multi-Objective Gray-Wolf Optimization for Attribute Reduction. Procedia Computer Science, 65, 623-632. doi: 10.1016/j.procs.2015.09.006

Faris, H., Aljarah, I., Al-Betar, M. & Mirjalili, S. (2017). Grey wolf optimizer: a review of recent variants and applications. Neural Computing and Applications, 30(2), 413-435.doi: 10.1007/s00521-017-3272-5.

Fouad, M.M., Hafez, A.I., Hassanien, A.E. & Snasel, V. (2015). Grey wolves optimizer-based localization approach in WSNs. In: 2015 11th international computer engineering conference (ICENCO). IEEE, pp 256–260.

Gao, Z.M. & Zhao J. (2019). An improved grey wolf optimization algorithm with variable weights. Computer Intelligence Neuroscience. https:// doi.org/10.1155/2019/2981282.

Glover, F. (1986). Future paths for integer programming and links to artificial intelligence, Computers & Operations Research, vol. 13, no. 5, pp. 533-549. Available: 10.1016/0305-0548(86)90048-1. https://doi.org/10.1016/0305-0548(86)90048-1.

Harifi, S., Mohammadzadeh, J., Khalilian, M. & Ebrahimnejad, S. (2020). Giza Pyramids Construction: an ancient-inspired metaheuristic algorithm for optimization. Evolutionary Intelligence. doi: 10.1007/s12065-020-00451-3.

Ihsan, R., Almufti, S., Ormani, B., Asaad, R. & Marqas, R. (2021). A Survey on Cat Swarm Optimization Algorithm. *Asian Journal of Research in Computer Science*, 22-32. doi: 10.9734/ajrcos/2021/v10i230237.

Jayabarathi, T., Raghunathan, T., Adarsh, B.R. & Suganthan, P.N. (2016). Economic dispatch using hybrid grey wolf optimizer. Energy 111:630–641.

Joshi, H. & Arora, S. (2017). Enhanced grey wolf optimisation algorithm for constrained optimisation problems. International Journal of Swarm Intelligence, 3(2/3), 126. doi: 10.1504/ijsi.2017.087871

Kohli, M. & Arora, S. (2018). Chaotic grey wolf optimization algorithm for constrained optimization problems. J Comput Design Eng 5(4):458–472.

Korayem, L., Khorsid, M. & Kassem, S.S. (2015) Using Grey Wolf algorithm to solve the capacitated vehicle routing problem. In: IOP conference series: materials science and engineering, vol 83, no 1. IOP Publishing, p 012014.

Kumar, V., Chhabra, J.K. & Kumar, D. (2017). Grey wolf algorithm based clustering technique. J Intell Syst 26(1):153–168.

Li, L., Sun, L., Kang, W., Guo, J, Han, C. & Li, S. (2016). Fuzzy multilevel image thresholding based on modified discrete grey wolf optimizer and local information aggregation. IEEE Access 4:6438–6450.

Li, S., & Wang, J. (2015). Dynamic Modeling of Steam Condenser and Design of PI Controller Based on Grey Wolf Optimizer. Mathematical Problems In Engineering, 2015, 1-9. doi: 10.1155/2015/120975

Liu, H., Hua, G., Yin, H. & Xu, Y. (2018). An intelligent grey wolf optimizer algorithm for distributed compressed sensing. Comput Intell Neurosci. https://doi.org/10.1155/2018/1723191.

Long, W., Jiao, J., Liang, X., & Tang, M. (2018). An exploration-enhanced grey wolf optimizer to solve high-dimensional numerical optimization. Engineering Applications Of Artificial Intelligence, 68, 63-80. doi: 10.1016/j.engappai.2017.10.024

Long, W., Wu, T., Cai, S., Liang, X., Jiao, J. & Xu, M. (2019). A novel grey wolf optimizer algorithm with refraction learning. IEEE Access 7:57805–57819.

Lu, C., Gao, L., Li, X. & Xiao, S. (2017). A hybrid multi-objective grey wolf optimizer for dynamic scheduling in a real-world welding industry. Engineering Applications Of Artificial Intelligence, 57, 61-79. doi: 10.1016/j.engappai.2016.10.013

Lu, C., Xiao, S., Li, X. & Gao, L. (2016). An effective multi-objective discrete grey wolf optimizer for a real-world scheduling problem in welding production. Advances In Engineering Software, 99, 161-176. doi: 10.1016/j.advengsoft.2016.06.004

Luo, Q., Zhang, S., Li, Z. & Zhou, Y. (2015). A novel complex-valued encoding grey wolf optimization algorithm. Algorithms 9(1):4.

Marashdih, A.W., Zaaba, Z.F. & Almufti, S.M. (2018). The Problems and Challenges of Infeasible Paths in Static Analysis.Int. J. Eng.Technol.2018, 7, 412–417.

Marqas, R., Almufti, S. & Asaad, R. (2019). Comparative study between elephant herding optimization (EHO) and U-turning ant colony optimization (U-TACO) in solving symmetric traveling salesman problem (STSP). Journal of Advanced Computer Science & Technology, 8(2), 32.doi: 10.14419/jacst.v8i2.29403.

Marqas, R., Almufti, S., Othman, P. & Abdulrahman, C. (2020). Evaluation of EHO, U-TACO and TS Metaheuristics algorithms in Solving TSP. Journal of XI'an University of Architecture & Technology, XII (IV).doi: 10.37896/jxat12.04/1062.

Medjahed, S., Ait Saadi, T., Benyettou, A. & Ouali, M. (2016). Gray Wolf Optimizer for hyperspectral band selection. Applied Soft Computing, 40, 178-186. doi: 10.1016/j.asoc.2015.09.045

Mirjalili, S. (2015). How effective is the Grey Wolf optimizer in training multi-layer perceptrons. Applied Intelligence, 43(1), 150-161. doi: 10.1007/s10489-014-0645-7

Mirjalili, S., Mirjalili, S. & Lewis, A. (2014). Grey Wolf Optimizer. Advances In Engineering Software, 69, 46-61. doi: 10.1016/j.advengsoft.2013.12.007

Mirjalili, S., Mirjalili, S.M. & Hatamlou, A. (2016). Multi-verse optimizer: a nature-inspired algorithm for global optimization. Neural Computer Application, 27(2):495–513.

Mittal, N., Singh, U. & Sohi, B. (2016). Modified Grey Wolf Optimizer for Global Engineering Optimization. Applied Computational Intelligence And Soft Computing, 2016, 1-16. doi: 10.1155/2016/7950348

Mohamed, A.A.A., El-Gaafary, A.A.M., Mohamed, Y.S. & Hemeida, A.M. (2015). Design static var compensator controller using artificial neural network optimized by modify grey wolf optimization. In: 2015 international joint conference on neural networks (IJCNN). IEEE, pp 1–7.

Mosavi, M.R., Khishe, M. & Ghamgosar, A. (2016). Classification of sonar data set using neural network trained by gray wolf optimization. Neural Network World 26(4):393.

Negi, G., Kumar, A., Pant, S. & Ram M., (2020). GWO: a review and applications, International Journal of System Assurance Engineering and Management, vol. 12, no. 1, pp. 1-8, Available: 10.1007/s13198-020-00995-8 [Accessed 2 August 2021].

Panda, M. & Das, B. (2019). Grey Wolf Optimizer and Its Applications: A Survey. Lecture Notes in Electrical Engineering, 179-194.doi: 10.1007/978-981-13-7091-5_17.

Parsian, A, Ramezani, M. & Ghadimi, N. (2017). A hybrid neural network-Gray Wolf optimization algorithm for melanoma detection. Biomed Res 28(8)

Saeed, V., Marqas, R. & Almufti, S. (2019). Taxonomy of bio-inspired optimization algorithms. Journal of Advanced Computer Science & Technology, 8(2), 23.doi: 10.14419/jacst.v8i2.29402.

Salim, B., Almufti, S. & Asaad, R. (2018). Review on Elephant Herding Optimization Algorithm Performance in Solving Optimization Problems. International Journal of Engineering & Technology 7: 6109-6114

Sanchez, D., Melin, P. & Castillo, O. (2017). A Grey Wolf Optimizer for Modular Granular Neural Networks for Human Recognition. Computational Intelligence and Neuroscience, 2017, 1-26. doi: 10.1155/2017/4180510

Shaban, A. & Almufti, S. (2018). U-Turning Ant Colony Algorithm for Solving Symmetric Traveling Salesman Problem. Academic Journal of Nawroz University, 7(4), 45-49. doi: 10.25007/ajnu.v6n4a270.

Singh, N & Singh S.B. (2017a). Hybrid algorithm of particle swarm optimization and grey wolf optimizer for improving convergence performance. J Appl Math. https://doi.org/10.1155/2018/ 1723191.

Singh, N. & Singh, S. (2017b). A novel hybrid GWO-SCA approach for optimization problems. *Engineering Science and Technology*, An International Journal, 20(6), 1586-1601. doi: 10.1016/j.jestch.2017.11.001

Song, H., Sulaiman, M. & Mohamed, M. (2014). An Application of Grey Wolf Optimizer for Solving Combined Economic Emission Dispatch Problems. International Review On Modelling And Simulations (IREMOS), 7(5), 838. doi: 10.15866/iremos.v7i5.2799

Song, X., Tang, L., Zhao, S., Zhang, X., Li, L., Huang, J. & Cai, W. (2015). Grey Wolf Optimizer for parameter estimation in surface waves. Soil Dynamics and Earthquake Engineering, 75, 147-157. doi: 10.1016/j.soildyn.2015.04.004

Tawhid, M.A., Ali, A.F. (2017). A Hybrid grey wolf optimizer and genetic algorithm for minimizing potential energy function. Memetic Computer 9(4):347–359

Tsai, P.W. & Dao, T.K., et al (2016). Robot path planning optimization based on multi objective grey wolf optimizer. In: International conference on genetic and evolutionary computing. Springer, pp 166–173

Vosooghifard, M. & Ebrahimpour, H. (2015). Applying grey wolf optimizer-based decision tree classifier for cancer classification on gene expression data. In: 2015 5th international conference on computer and knowledge engineering (ICCKE). IEEE, pp 147–151.

Wen, L., Dongquan, Z. & Songjin, X.U. (2015). Improved Grey Wolf Optimization algorithm for constrained optimization problem. Journal Computer Applications 35(9):2590–2595.

Wong, L.I., Sulaiman, M.H., Mohamed, M.R. & Hong, M.S. (2014). Grey wolf optimizer for solving economic dispatch problems. In: 2014 IEEE international conference on power and energy (PECon). IEEE, pp 150–154.

Yadav, S., Verma, S.K. & Nagar, S.K. (2016). Optimized pid controller for magnetic levitation system. IFAC-PapersOnLine 49(1):778–782.

Yahya Zebari, A., Almufti, S. & Khalid Omer, H. (2019). A comparative study of particle swarm optimization and genetic algorithm. Journal of Advanced Computer Science & Technology, 8(2), 40. doi: 10.14419/jacst.v8i2.29401.

Yahya Zebari, A.M., Almufti, S. & Mohammed Abdulrahman, C. (2020). Bat algorithm (BA): review, applications and modifications. International Journal of Scientific World, 8(1), 1. doi: 10.14419/ijsw.v8i1.30120.

Yamany, W., Emary, E. & Hassanien, A.E. (2016). New rough set attribute reduction algorithm based on grey wolf optimization. In: The 1st international conference on advanced intelligent system and informatics (AISI2015), November 28–30, 2015, Beni Suef, Egypt. Springer, pp 241–251.

Yang, H. (2015). A hybrid clustering algorithm based on Grey Wolf Optimizer and K-means algorithm. Journal of Jiangxi University of Science and Technology.

Yassien, E., Masadeh, R., Alzaqebah, A., Shaheen (2017) A Grey Wolf optimization applied to the 0/1 knapsack problem. International Journal of Computer Applications (0975–8887) 169(5).

Zhang, S. & Zhou, Y. (2015). Grey Wolf Optimizer Based on Powell Local Optimization Method for Clustering Analysis. Discrete Dynamics In Nature And Society, 2015, 1-17. doi: 10.1155/2015/481360